Performance analysis of partially coherent SSD systems

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The performance analysis of a system employing co-ordinate interleaving and constellation rotation, over Nakagami-m fading channels, in the presence of phase noise as well as additive white Gaussian noise, is presented. An upper bound for the average probability of bit error for M-ary phase shift keying is derived. It is shown that the derived upper bound is tight for high signal-to-noise ratios and in the presence of phase noise the optimum rotation angle does not change. Furthermore, it is shown that a system employing co-ordinate interleaving and constellation rotation is more robust against phase estimation errors.

Introduction: Coherent detection of M-ary phase shift keying (MPSK) signals requires the generation or extraction of a local carrier phase reference. In most applications a perfect local reference is not possible and hence the noise associated with the carrier leads to degradation of system performance [1, 2]. It is well known that diversity can lead to a significant performance improvement of transmission over fading channels. An attractive option is signal space diversity (SSD) (also known as co-ordinate interleaving) where multidimensional signal constellations are used and the rotated components of each signal constellation point are transmitted over independent fading channels.

The analysis carried out on systems employing SSD in the literature, e.g., [3, 4], is under the assumption that a perfect phase reference is available at the receiver for demodulation. In practice, however, this is not possible. In this Letter, we present closed form analytical expressions of the union bound of the average probability of bit error (P_{b}) for MPSK signal constellations over Nakagami-m fading channels with phase noise. We show that the derived bound is tight for higher SNRs. Furthermore, it is shown that the optimum rotation angles for a system employing SSD does not change under the influence of phase noise.

System model: A conventional MPSK signal constellation is denoted by $S_{m} = \{ s_{i} = e^{j2\pi i/M}; i = 0, 1, \ldots, M-1 \}$, where the energy has been constrained to unity and each symbol corresponds to $m = \log M$ bits. Anticlockwise rotation over an angle $\theta$ leads to the constellation $S_{m}^{\theta} = \{ s_{i} = e^{j2\pi i/m}; i = 0, 1, \ldots, M-1 \}$. The symbol mapper can be represented by a one-to-one mapping function $\nu$: $\{ 0, 1 \}^{\nu} \rightarrow S_{m}^{\theta}$, $s = \nu(b)$, where $b = (b_{1}, \ldots, b_{\nu})$. $b_{i} \in \{ 0, 1 \}$ represents the binary sequence and $s$ is chosen from the set $S_{m}^{\theta}$ consisting of $M$ complex signal points. Let $\eta$ and $\mu$ represent the $I$ and $Q$ interleavers, resulting in sequences $x = \eta(x) = (x_{1}, x_{2}, \ldots, x_{\nu})$ and $y = \mu(y) = (y_{1}, y_{2}, \ldots, y_{\nu})$, respectively. As we have two orthogonal channels, let the squared Euclidean distances between two different signal constellation points in $I$ and $Q$ directions be represented by $d_{i}^{2} \text{ and } d_{i}^{2}$, respectively. The distances are given as

$$d_{i}^{2} = (\cos(\theta_{i}) + \cos(\theta_{i} + \theta))^{2}$$

$$d_{i}^{2} = (\sin(\theta_{i}) + \sin(\theta_{i} + \theta))^{2}$$

where $\theta_{i}$ and $\theta_{i}$ represent the phases of the two signal constellation points under consideration, respectively.

Plane fading channel further perturbed by complex additive white Gaussian noise (AWGN) with a one-sided power spectral density of $N_{0}$. The received signal is demodulated by a partially coherent receiver using phase-locked loops (PLL) for phase recovery. Parallel coherence implies that the reference phase at the receiver contains a random phase error. Furthermore, it is assumed that the PLL SNR is much greater than $\gamma$, where $\gamma = E_{S}/N_{0}$ is the average received SNR per bit. This assumption implies that the cosine of the phase estimate can be replaced by a deterministic variable (expected value) in the analysis [1]. This approximation technique is also referred to as linear approximation as it is exact when the function whose expectation is to be approximated is linear, regardless of the variance [1]. A partially coherent receiver provides (via a PLL) a complex reference signal $e^{-j\phi}$ whose phase $\phi$ is an estimate of the unknown channel phase $\phi_{c}$. The phase estimation error $\phi_{e} = \phi - \phi_{c}$ follows the Tikhonov distribution under the normal operating conditions, i.e., when the PLLs are

in lock [1]. The Tikhonov probability density function (pdf) is $p(\phi_{e}) = e^{-(\phi_{e})^{2}/2\pi\rho\sigma^{2}}$, where $I_{s}(\cdot)$ is the $k$th-order modified Bessel function of the first kind and $\rho$ is the tracking PLL SNR. After phase removal, the received samples are deinterleaved and the receiver performs a maximum likelihood (ML) detection.

Performance analysis of SSD system over Nakagami-m fading channel: The pdf of the Nakagami-m distribution for $m \geq 0.5$ is given as [5]

$$p(\alpha) = \frac{2m^{m/2} \alpha^{m-1}}{\Gamma[m]} e^{-m \alpha^{2}}, \quad \alpha \geq 0$$

with $c(\alpha) = 1$, $\Gamma[1]$ is the gamma function, and $m$ is the shape parameter. A standard approach of evaluating the error probability, $P_{b}$, of a signal set $S_{m}^{\theta}$ is based on the union bound [5]

$$P_{b} \leq \frac{1}{m^{2}} \sum_{s \in S_{m}^{\theta}} \sum_{s \in S_{m}^{\theta}} a(s, \tilde{s})P(s \rightarrow \tilde{s})$$

where $P(s \rightarrow \tilde{s})$ is the pairwise error probability (PEP) that the receiver estimated $\tilde{s}$ when $s$ was transmitted; given that $s$ and $\tilde{s}$ are the only two signal constellation points under consideration. $a(s, \tilde{s})$ represents the Hamming distance between the bit sequences of $s$ and $\til{s}$ under consideration.

Let $a_{1}$ and $a_{2}$ be Nakagami-m distributed random variables with the pdf given by (2). To calculate $P_{b}$ for a system employing co-ordinate interleaving affected by phase noise and fading, the conditional PEP needs to be averaged over the fading static as well as the phase noise. Using the linear approximation [1], the averaging can be simplified as

$$P(s \rightarrow \tilde{s}) = \int_{0}^{\infty} \int_{0}^{\infty} Q\left( \sqrt{\frac{(\sin^{2}(\theta) + \sin^{2}(\theta)) + \sin^{2}(\theta)}{2}} \right)^{m} d\theta d\phi$$

$$P_{b} \leq \frac{1}{m^{2}} \sum_{s \in S_{m}^{\theta}} \sum_{s \in S_{m}^{\theta}} \int_{0}^{\infty} \int_{0}^{\infty} Q\left( \sqrt{\frac{(\sin^{2}(\theta) + \sin^{2}(\theta)) + \sin^{2}(\theta)}{2}} \right)^{m} d\theta d\phi$$

$$P(s \rightarrow \tilde{s}) = \frac{1}{2} \frac{d_{i}^{2}}{2(2d_{i}^{2} - C)} \left( \frac{\frac{\sin^{2}(\theta)}{2} + \frac{\sin^{2}(\theta)}{2}}{\gamma} \right)$$

$$+ \frac{C - d_{i}^{2}}{2(2d_{i}^{2} - C)} \left( \frac{\frac{\sin^{2}(\theta)}{2} + \frac{\sin^{2}(\theta)}{2}}{\gamma} \right)$$

where $C = d_{i}^{2} + d_{i}^{2}$. Results and discussion: In the subsequent analysis we take QPSK signal constellation as an example signal constellation from MPSK and, also, we take $m = 1$, i.e. Rayleigh fading, as an example fading environment. (a) Optimal theta: The optimum rotation angle $\theta$ for $P_{b}$ may be found by minimising (3). In (3), we need to take the derivative of (4).

Using the derivative and (3) an optimum angle of $\theta = 17.6^\circ$ is calculated by employing the steepest descent algorithm. The calculated optimum rotation angle is the same as for a system with perfect phase estimates [4] also shown by Fig. 1. This Figure also shows that the derived bound is tight and is in good agreement with the simulation results at high SNRs.
Fig. 1 Average probability of error $P_b$ and $P_{UB}$ against rotational angle $\theta$ of SSD system using QPSK signal constellation over Rayleigh fading channel at $E_b/N_0 = 15$ dB in presence of phase noise

Gray signal constellation mapping used

(b) Phase error effect: Fig. 2 shows the degradation in the performance of an SSD system when the phase noise is taken into account. It is seen that the performance degrades more when the PLL SNR, $\rho$, is held constant while the system SNR, $\gamma$, is increased. This effect is not realistic and can be considered as a worst-case scenario. Even under such a scenario when the PLL is generating more noise relative to the rest of the receiver, the SSD system has a better performance than a conventional QPSK system. In a practical system, however, the second-order PLL's $\rho$ is proportional to $\gamma$ and as $\gamma$ increases PLL performs better and better [1, 2]. A more realistic assignment is as [1],

$$\rho = \tau\gamma$$  \hspace{1cm} (6)

where $\tau$ is a constant. In Fig. 2, for instance, a curve is included based on (6) where $\tau = 5$ dB is used, which shows the performance gain of an SSD system over a conventional QPSK system ($P_b = 1/2\{1 - \sqrt{\gamma/(1 + \gamma)}\}$). Overall, in all cases, the SSD system results in a better performance in terms of $P_b$ compared to a conventional QPSK system in the presence of phase noise.

Fig. 2 Average probability of error $P_b$ of SSD and conventional system using QPSK signal constellation over Rayleigh fading channel in presence of phase noise

Gray signal constellation mapping used

Conclusions: We have investigated the performance of a system employing SSD in the presence of phase noise. An expression for the upper bound of $P_b$ is derived for $M$-PSK signal constellations. The new bound is tight for the entire range of the constellation rotation angle at high SNR. We have shown that the optimum rotation angle does not change under the influence of phase noise when using the linear approximation [1]. Furthermore, we have shown that the SSD system is more robust against phase noise than a conventional $M$-PSK system.

References